

Algorithms for Exploring the Tree of Numerical Semigroups

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Abstract: There have been many efforts to count numerical semigroups by the genus and by the Frobenius number. It is known that the number of semigroups of each genus grows asymptotically with the genus like the Fibonacci numbers and that the number of semigroups of each Frobenius number grows asymptotically in such a way that each number doubles the previous but one. We will provide a formula for the number of numerical semigroups of each Frobenius number, genus, and multiplicity, under the assumption that the multiplicity m and the Frobenius number F satisfy $m \geq \frac{F+1}{3}$. This formula gives, under the required restriction, a multiparameter exact version of the increasing behaviours just mentioned.

We will introduce the unleaved tree of numerical semigroups of a given genus and the Frobenius-leaf-discriminating tree of a given Frobenius number. The first tree is characterized by the fact that its leaves are exactly the semigroups of a given genus, while the second tree is characterized by the fact that its leaves are exactly the semigroups of a given Frobenius number.

We will finally present two adaptations of the seeds algorithm to explore both the unleaved tree and the Frobenius-leaf-discriminating tree. For this purpose we adapted the recursive descending algorithm for trimming the tree exactly at those nodes with no descendants with a given genus, in the first case, or with no descendants with a given Frobenius number, in the second case.

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