Covarieties and their generalizations

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Abstract: In this talk, with the objective of contributing to the resolution of the three foundational problems in semigroup theory: Frobenius Problem([8]), Wilf's conjecture([3], [4]) and Bras-Amorós conjecture ([1], [2]). We shall present covarieties (see [5]), that is, a nonempty family $\mathscr{C}$ of numerical semigroups that fulfills the following conditions: there is the minimum of $\mathscr{C}$, the intersection of two elements of $\mathscr{C}$ is again an element of $\mathscr{C}$ and $S\backslash\{\mathrm{m}(S)\} \in \mathscr{C}$ for all $S \in \mathscr{C}$ such that $S \neq \min(\mathscr{C})$.

Additionally, we introduce ratio-covarieties (see [7]), that is a family $\mathscr{R}$ of numerical semigroups that has a minimum, denoted by $\min(\mathscr{R})$, is closed under intersection, and if $S \in \mathscr{R}$ and $S \neq \min(\mathscr{R})$, then $S\backslash\{\mathrm{r}(S)\} \in \mathscr{R}$, where $\mathrm{r}(S)$ denotes the ratio of S. and establish that these two major families can be generalized. We will show that $\mathrm{MED}(F, m) = \{S \mid S$ is a numerical semigroup with maximal embedding dimension, Frobenius number F and multiplicity$\}$ is a ratio-covariety. As a consequence, we present two algorithms: one that computes $\mathrm{MED}(F, m)$ and another one that calculates the elements of $\mathrm{MED}(F, m)$ with a given genus.

We illustrate these concepts through examples, showing that these families can be ordered as trees. Algorithms to obtain them, based on the criterion of a fixed notable element, will also be presented.

Finally, we will see that there exists the generalization of this families of numerical semigroups(see [6]).

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References

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