

Sally type Numerical Semigroups and Semigroup Rings

HEMA SRINIVASAN

University of Missouri

Abstract: A numerical semigroup of multiplicity e and width $e - 1$ is a Sally type numerical semigroup and the corresponding ring is the Sally type numerical semigroup ring. The name is inspired by July Sally's paper on "Good embedding for Gorenstein Rings", where a good embedding is one where the associated graded ring is guaranteed to be Cohen Macaulay. She finishes this problem with a proof ending in a numerical semigroup of multiplicity e and embedding dimension $e - 1$ and computes the minimal generators for the tangent cone. In this talk, we will describe Sally type numerical semigroups of embedding dimension $e - 2$, construct minimal resolution for some of them. We will discuss various properties of and offer some conjectures for the structure of Betti numbers of these semigroup rings.

These results are from a WICA project with Saipriya Dubey, Kriti Goel, Nil Sahin and Srishti Singh.

July Sally showed that a semigroup minimally generated by $e, e + 1, e + 3, \dots, 2e - 1$ is Gorenstein. This is a Sally type semigroup with exactly two consecutive numbers missing. We will show that for any k , there is exactly one of the Sally type semigroups of multiplicity e with exactly k consecutive numbers missing is Gorenstein and construct the minimal resolution of the semigroup ring. Their generators and resolution look very similar to the ones defined by arithmetic sequence although they are clearly not defined by an arithmetic sequence.

A Sallytype semigroup of multiplicity e is minimally generated by a subset of an arithmetic sequence $e, e + 1, \dots, 2e - 1$, of length $e - 1$ and common difference 1. We can consider Sallytype d semigroups which are minimally generated by a subset of an arithmetic sequence of length $e - 1$ and common difference d . We will explain some of our initial results in this direction.